MI-0200

Rectangular Combined Function Magnets

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Introduction

The Recycler Ring, which is being designed for the Main Injector tunnel, will use permanent magnet combined function magnets. In the terminology of **MAD** these will be *rbends* and not *sbends* such as were used in the Fermilab Booster. The difference between the two type of magnets is that in a *sbend* the magnet is bent so that a particle on the closed orbit travels along the nominal center line of the magnet. [1] In a *rbend* the trajectory of the particle is not along the magnet center line. In a dipole this displacement makes no difference but in a *rbend* with a gradient it means that the field, seen by a particle, varies as the particle moves through the magnet.

In this note I will describe the results of calculations of the trajectory through a *rbend* and suggest a way of minimizing the effect of the non-uniformity of the field on the particles in the beam.

In Appendix I will derive some simple results on the properties of the reference trajectory.

The topics in this note are also discussed, with similar results, in MI-0195 and MI-0196 by S.D. Holmes.

[1] I denote as the center line as the line in the magnet the line to which the magnet measurements are referenced.

Closed Orbit

The closed orbit in an accelerator is the trajectory of a particle which repeats, closes on itself, after one transversal of the machine. It is easy to construct the closed orbit if:

- the particle enters and leaves the bend magnets at the same distance from the center line of the magnet; and if
- the net bend of the trajectory is just the nominal bend of the magnet.

The trajectory of a particle has been traced through a *rbend* with a gradient field and the initial position and angle, relative to the center line of the magnet, determined such that the conditions above are satisfied.

The resulting trajectory is not a circle, because the magnetic field is not uniform, but differs from a circular trajectory which satisfies the above conditions, by no more the $\approx 50\mu m$.(figure 1) [2]

In addition to the dipole and quadrupole moments, the Recycler magnets will have built in sextupole components, to correct the chromaticity, and higher order multipoles resulting from production tolerances. These higher order multipoles can change both the reference trajectory and the effective quadrupole and sextupole and components of the magnets.

If the sextupole and other high order multipoles are included in the description of the magnetic field, the location of the initial position of the reference orbit changes by less that $10\mu m$.

Feed Down of the High Order Multipoles

It is fairly easy to show [3] that for a circular orbit which satisfies the two conditions listed above, that the feed down, or contribution from the b_n multipole into the next lower multipole, b_{n-1}, is zero. Thus the quadrupole does not change the dipole strength of the magnet [4] and the sextupole does not change the focusing strength of the magnet. Since the actual reference

[2] The circle does not represent a solution to the equations of motion.

[3] See the appendix.

[4] Nor should it, since the bend angle is the same as it would be in the absence of the gradient by construction.

trajectory differs from a circle by a small distance we would expect these conclusions to be true for the actual trajectory.

A calculation of the feed down effect, using the values for the normal multipoles found in MI-0170. The results are plotted in figure 2 and figure 3. It is clear that for the reference trajectory the change in b_0 and b_1 is very small. The change in the other multipole moments is also small but not negligible compared to their original values.

Conclusions

If the Recycler Ring is aligned so that the beam enters the bends at the location below the magnet center line given in table I then both the bend strength and the focusing strength of the magnet, as seen by a particle on the reference trajectory, will be the nominal values given in the MAD file.

Magnet Type	Displacement From Midline (mm)
Long Focusing	-7.44
Long Defocusing	-7.39
Short Focusing	-3.30
Short Defocusing	-3.28

Appendix

Derivation of the Initial Point of the Reference Trajectory

Consider a particle at a point (x, y) in a magnetic field $\mathbf{B}$. In traversing a distance dl the direction of the particle's motion will rotate an amount $\Delta\theta$ such that

$$\Delta\theta = \mathbf{B}dl/[B\rho]$$

If I assume that the reference trajectory is a circle which satisfies the two conditions imposed above, I can replace dl with $Rd\phi$ where $R = [B\rho]/\mathbf{B_0}$ is the radius of the circle.(figure 4).

We will now integrate $\Delta\theta$ from $\phi = -\theta_0/2$ to $+\theta_0/2$ where θ_0 is the nominal bend angle of the magnet.

In the case in which $\mathbf{B}$ is constant we of course recover the fact that $\Delta\theta = \theta_0$.

In the case of interest, with a quadrupole field in the magnet,

$$\mathbf{B} = \mathbf{B_0} + \mathbf{B}'(y - h)$$

where h is the distance along y from the entrance point of the trajectory to the midline of the magnet.

Using this expression for $\mathbf{B}$ and integrating along a circular trajectory that satisfies the first of the above conditions

$$\Delta\theta = (R/[B\rho]) \cdot \int_{-\theta_0/2}^{+\theta_0/2} (\mathbf{B_0} + \mathbf{B}'(y - h))d\phi$$

$$\Delta\theta = (R/[B\rho]) \cdot (\mathbf{B_0} \cdot \theta_0 + \mathbf{B}' \cdot (R \cdot 2\sin(\theta_0/2) - \theta_0 \cdot (R \cdot \cos(\theta_0/2) + h))$$

Since $R = [B\rho]/\mathbf{B_0}$

$$\Delta\theta = \theta_0 + \mathbf{B}' \cdot (2R \cdot \sin(\theta_0/2) - \theta_0 \cdot (R \cdot \cos(\theta_0/2) + h)$$

then to satisfy the second condition

$$\mathbf{B}' \cdot (2R \cdot \sin(\theta_0/2) - \theta_0 \cdot (R \cdot \cos(\theta_0/2) + h)) = 0$$

Note that in this approximation, i.e., a circular orbit, the value of h does not depend on the gradient $\mathbf{B}'$ and depends only on the bend angle and R.

Solving for h we find

$$h = (2R/\theta_0)(\sin(\theta_0/2) - \theta_0/2 \cdot \cos(\theta_0/2))$$

To order θ_0^3, $h = R \cdot \theta_0^2/12$. This should be compared with half the sagitta $s/2 = R \cdot \theta_0^2/16$. The difference $|h| - (s/2) = R \cdot \theta_0^2/48$ which for the long Recycler ring magnets is $\approx 1.9mm$.

Feed down of the Multipoles

The feed down of the of the n^{th} multipole into the $n-1$ multipole is given by

$$\delta b_{n-1} = n \cdot b_n \cdot R \cdot \int (y - h)d\phi$$

As shown above, to satisfy our two conditions on the trajectory and in the approximation of a circular trajectory, the above integral vanishes. Thus along the trajectory which satisfies the conditions the gradient does not feed down into the b_0 and the sextupole does not change the focusing of the magnets.

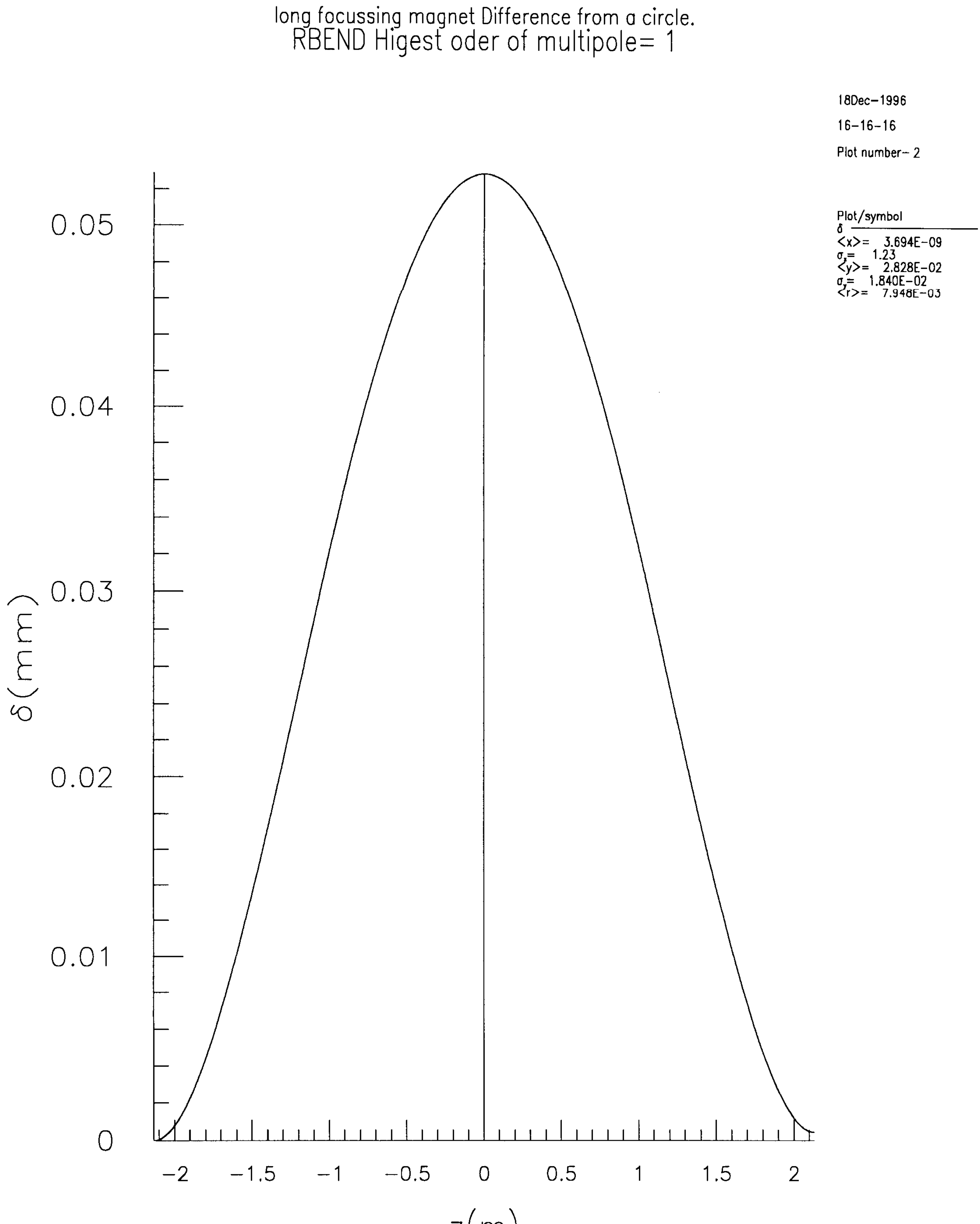
long focussing magnet Difference from a circle.
RBEND Higest oder of multipole= 1
18Dec-1996
16-16-16
Plot number- 2
Plot/symbol
δ
<x>= 3.694E-09
σx= 1.23
<y>= 2.828E-02
σy= 1.840E-02
<r>= 7.948E-03
0.05
0.04
0.03
0.02
0.01
0
δ(mm)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
z(m)

Figure 1

long focussing magnet Variation of the Multipoles With Initial Position
RBEND

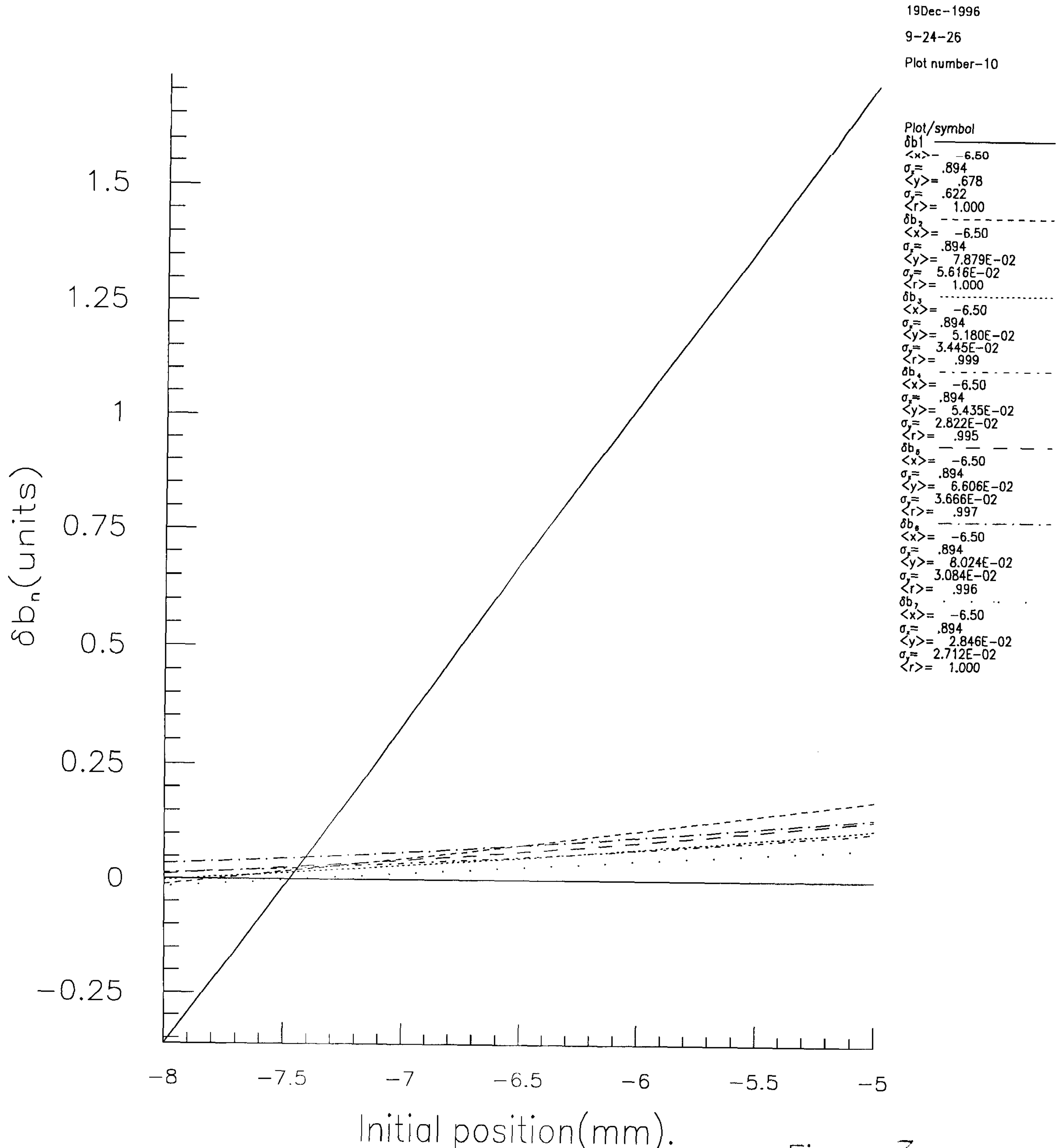

Figure 3

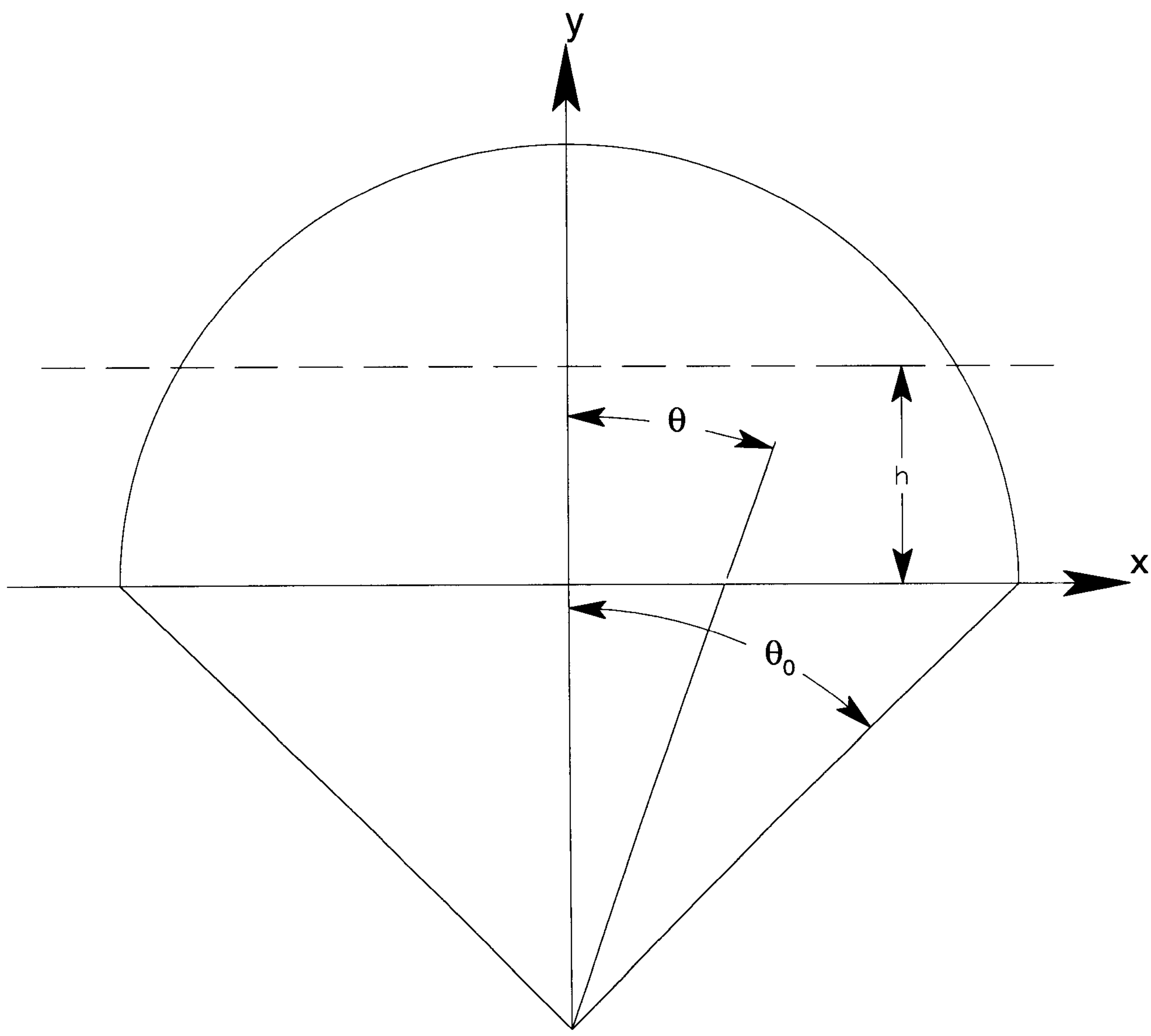

Figure 4